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LETTER TO THE EDITOR

Magnetic excitations in some generalised Fibonacci layered structures

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Abstract. Long-wavelength magnetic excitations in aperiodic layered structures consisting of alternating magnetic and non-magnetic layers are investigated. Real and periodic boundary conditions are used. Spin-wave spectra for Fibonacci superlattices with copper mean and golden mean are compared.

Recent advances in experimental techniques have inspired studies [1–16] of electronic, optical, acoustic, superconducting and magnetic properties of quasi-periodic layered systems. Until our work [17–19] and the work of Riklund *et al* [8] and Fujita and Machida [9], studies of one-dimensional quasi-crystals were based mainly on the Fibonacci superlattice with the golden mean. We found [17–19] that the dynamical trace maps of generalised Fibonacci superlattices are quite different to that of the Fibonacci lattice with the golden mean. Here we partially answer the question of whether their physical properties are also different. In our generalisation [17, 18] it was assumed that a binary Fibonacci superlattice is formed by distributing two types of building blocks A and B according to the inflation scheme

$$S_{L+1} = S_L^m S_{L-1}^n \tag{1}$$

where $S_0 \equiv \mathrm{B}$, $S_1 \equiv \mathrm{A}$, and m and n are integers. This inflation scheme is equivalent to a generalised substitution rule $\mathrm{A} \rightarrow \mathrm{A}^m\mathrm{B}^n$, $\mathrm{B} \rightarrow \mathrm{A}$, where A^m represents a string of m As. The total number of blocks of types A and B in S_L is equal to the generalised Fibonacci number F_L defined by the recurrence relation $F_L = mF_{L-1} + nF_{L-2}$, $F_0 = F_1 = 1$. Then the limit value of the ratio of two subsequent F_Ls is

$$\sigma = \lim_{L\to\infty} F_L/F_{L-1} = \tfrac{1}{2}[m + (m^2 + 4n)^{1/2}].$$

Setting $m = n = 1$ gives the standard Fibonacci sequence with the *golden mean* $\sigma = \sigma_g = \frac{1}{2}(1 + \sqrt{5})$. The generalised Fibonacci superlattices with the *silver mean* $\sigma_s(m = 2, n = 1) = 1 + \sqrt{2}$, *copper mean* $\sigma_c(m = 1, n = 2) = 2$ and *nickel mean* $\sigma_n(m = 1, n = 3) = \frac{1}{2}(1 + \sqrt{13})$ have recently been studied [17, 18].

Let us denote by $N_L(\mathrm{A})$ and $N_L(\mathrm{B})$ the number of blocks of types A and B, respectively, contained in S_L, i.e. $F_L = N_L(\mathrm{A}) + N_L(\mathrm{B})$. The following simple relations are true in the case of the copper mean: $N_L(\mathrm{A}) = N_L(\mathrm{B}) \mp 1$, $N_L(\mathrm{A}) = 2N_{L-1}(\mathrm{A}) \mp 1$, $F_L = 2F_{L-1} \pm 1$, where the upper signs hold for L even and the lower ones for L odd. These

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relations differ from those for the Thue–Morse sequence [8] only by the presence of the ∓ 1 and ± 1 terms. While the generalised Fibonacci sequence with the copper mean is a genuinely aperiodic sequence, it possesses some 'residual symmetries' (similar to the Thue–Morse lattice) reminiscent of a periodic arrangement of A and B blocks. In the copper mean lattice the B blocks occur always in pairs separated by one, two or three A blocks. It is interesting to note that the copper mean lattice can be transformed to the periodic lattice AABBAABB . . . by local rearrangements of some A blocks. Furthermore, it can be seen that, by removing the last block, the stack for odd L has a mirror symmetry with respect to its centre. Recently, Fujita and Machida [9] have mentioned the substitution rule A $\rightarrow$ ABB, B $\rightarrow$ A, which is equivalent to what we call the copper mean case. However, the authors did not do any calculations for this case. The mean purpose of this Letter is to investigate the spin-wave spectra in the magnetostatic limit for the superlattice with the copper mean and to compare them with the results for the case with the golden mean [16]. In our calculations we will use the transfer matrix method with both the *real* (*rigid*) and *periodic* boundary conditions as opposed to only periodic boundary conditions used in [16].

In the present study the two building blocks A and B are assumed to be composed of a magnetic thin film of thickness l_{Am} or l_{Bm} followed by a non-magnetic film of thickness l_{An} or l_{Bn}. To get a truly aperiodic system, the inequalities $l_{\mathrm{Am}} \neq l_{\mathrm{Bm}}$ and/or $l_{\mathrm{An}} \neq l_{\mathrm{Bn}}$ must hold. Magnetic excitations are considered within the framework of the macroscopic theory of spin waves. We ignore spin relation, magnetic anisotropy and exchange interaction [15]. The basic theory for the problem is given in [15] and [16]. We investigate the case where the magnetic field $\boldsymbol{H}_0$ and the saturation magnetisation $\boldsymbol{M}_{\mathrm{s}}$ are parallel to the interfaces between the layers. The coordinate system is oriented such that the y direction is normal to the interfaces and the z direction is parallel to the applied field. In the magnetostatic limit, the demagnetising field $\boldsymbol{h}$ corresponding to a particular eigenmode with circular frequency ω (often measured in units of magnetic field, $\Omega = \omega/\gamma$) can be expressed in terms of magnetic potential, φ_{M}, such that $\boldsymbol{h} = -\nabla\varphi_{\mathrm{M}}$. Because of the translational invariance in the x and z directions, magnetic potential has the form $\varphi_{\mathrm{M}}(\boldsymbol{r}) = \exp[\mathrm{i}q(x\cos\theta + z\sin\theta)]\,\psi_{\mathrm{M}}(y)$, where q is the magnitude of the wavevector along the interface plane, and θ is the angle between this wavevector and the x axis. Here $\psi_{\mathrm{M}}(y) = A_{ij}\exp[\alpha_j(y - y_i)] + B_{ij}\exp[-\alpha_j(y - y_i)]$; $y_i < y < y_{i+1}$, where i enumerates the blocks in the Fibonacci sequence, $i = 1, 2, \ldots, F_L$, and j is the index of films inside the corresponding block ($j \neq \mathrm{m}$ for a magnetic film and $j \equiv \mathrm{n}$ for a non-magnetic film); y_i denotes the position of the interface between the $(i-1)$th and ith blocks, $\alpha_{\mathrm{n}} = q$ while α_{m} depends [16] on q, θ, Ω, H_0 and M_{s}. The relations between the constants A_{ij} and B_{ij} in the neighbouring films can be written in the transfer matrix form

$$\begin{pmatrix} A_{in} \\ B_{in} \end{pmatrix} = \mathbf{T}_{im}\begin{pmatrix} A_{im} \\ B_{im} \end{pmatrix} \qquad \begin{pmatrix} A_{i+1,\mathrm{m}} \\ B_{i+1,\mathrm{m}} \end{pmatrix} = \mathbf{T}_{in}\begin{pmatrix} A_{in} \\ B_{in} \end{pmatrix} \qquad \begin{pmatrix} A_{i+1,\mathrm{m}} \\ B_{i+1,\mathrm{m}} \end{pmatrix} = \mathbf{T}_{i}\begin{pmatrix} A_{im} \\ B_{im} \end{pmatrix} \tag{2}$$

where the 2×2 matrices $\mathbf{T}_{im}$ and $\mathbf{T}_{in}$ can be expressed [16] in terms of α_{n}, α_{m}, θ, film thicknesses l_{ij} and the components of a frequency-dependent tensor. In (2) $\mathbf{T}_i \equiv \mathbf{T}_{in}\mathbf{T}_{im}$ is the transfer matrix for the whole ith block. The stack S_L extends in the y direction from $y_{\mathrm{left}} \equiv y_1$ to $y_{\mathrm{right}} \equiv y_{F_L+1}$. In accordance with (1), the full transfer matrix $\mathbf{M}_L = \mathbf{T}_{F_L}\mathbf{T}_{F_L-1}\ldots\mathbf{T}_1$ of this stack satisfies the recursion relation

$$\mathbf{M}_{L+1} = \mathbf{M}_{L-1}^{n}\mathbf{M}_{L}^{m} \qquad \mathbf{M}_0 \equiv \mathbf{T}_{\mathrm{B}} \qquad \mathbf{M}_1 \equiv \mathbf{T}_{\mathrm{A}}. \tag{3}$$

Finally, it is necessary to specify some boundary conditions. One choice is the periodic

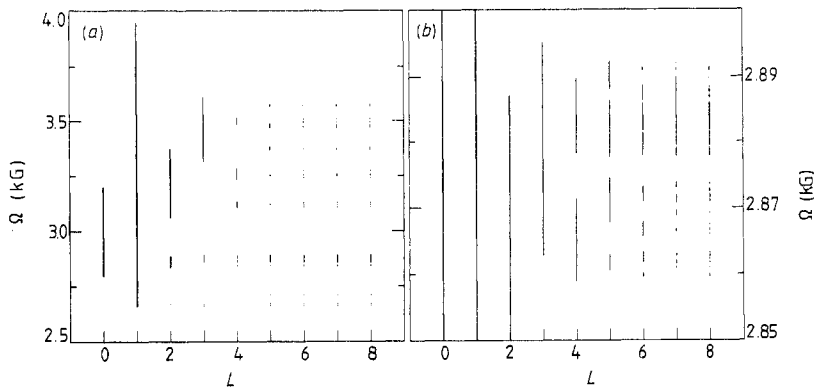

Figure 1. Spin-wave spectra with periodic boundary conditions for the copper mean lattice. L denotes the number of generations. Each block is made of Ni (magnetic) and Mo (non-magnetic) films. The following parameters were used: $M_s = 480$ G, $H_0 = 1000$ G; $ql_{Am} = 0.1$, $ql_{An} = 0.06$, $ql_{Bm} = 0.1$, $ql_{Bn} = 1.0$, $\theta = 0.0$. Allowed bands are represented by vertical line segments. (*a*) Full spectra, (*b*) magnification of the region containing the quasi-continuous cluster of bands.

boundary conditions [16] for A_{im} and B_{im} which imply the condition $-1 \leqslant X_L \leqslant 1$, where $X_L \equiv \frac{1}{2}\mathrm{Tr}(\mathbf{M}_L)$. The allowed eigenfrequencies Ω satisfying this condition span continuous bands. Since the matrices $\mathbf{M}_L$ are unimodular, one can work, as in the case of electronic states [3, 17], directly with the recursion relations for the traces X_L [17].

The golden mean case with the periodic boundary conditions is well illustrated in the work of Xiong [16],† for a Ni–Mo system with plots of allowed bands as a function of layer thicknesses, applied field and wavevector. In figure 1(*a*), we present the copper mean equivalents of the golden mean results of figure 1 of [16]. Unlike in the golden mean case, the spectrum of the copper mean lattice is not Cantor-like in its whole range. In the lower part of the spectra of figure 1(*a*) there is an almost continuous cluster of bands (extending from $\Omega = 2878.2$ to 2885.6 G) which is more clearly shown in figure 1(*b*). We have verified that the total width of allowed bands in this cluster exceeds the total width of gaps for up to at least 20 generations (699051 blocks). The existence of such quasi-continuous clusters is related to the properties of the dynamical trace map of the copper mean lattice as will be shown elsewhere. The difference in the band structures of these two cases was expected since their dynamical trace maps are distinct. The trace map for the golden mean lattice is volume-preserving at all points, while it is not so for the copper mean lattice [20].

For a single stack S_L, a more suitable choice is the real boundary conditions which also allow manifestations of possible surface effects that are neglected in the periodic boundary conditions. We assume that the stack S_L is embedded in an arbitrary non-magnetic medium. Thus on the *left* and *right* of the stack, $\psi_m(y)$ is given by $\psi_M(y) = A_{left}\exp(\alpha_n y); y < y_{left} = 0$, $\psi_M(y) = B_{right}\exp[-\alpha_n(y - y_{right})]; y > y_{right}$. The boundary conditions at $y = y_{left}$ and $y = y_{right}$ can be expressed in the form

$$\begin{pmatrix} A_{1m} \\ B_{1m} \end{pmatrix} = \mathbf{T}_{left} \begin{pmatrix} A_{left} \\ 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 0 \\ B_{right} \end{pmatrix} = \mathbf{T}_{right} \begin{pmatrix} A_{F_L+1,m} \\ B_{F_L+1,m} \end{pmatrix} \tag{4}$$

† It seems that there are misprints in the values of ω along the vertical axis of all the figures in [16], and the caption of figure 1 should contain $l_{Bm} = 0.1$ instead of $l_{Bm} = 0.01$.

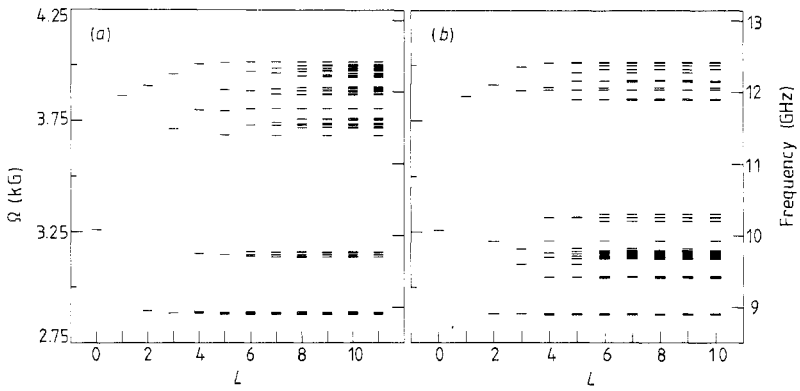

Figure 2. Spin-wave spectra with real boundary conditions for the same parameters as in figure 1, except for $ql_{Am} = 1.0$, $ql_{An} = 0.25$, $ql_{Bm} = 0.25$, $ql_{Bn} = 1.0$. The discrete eigenfrequencies are marked by short horizontal lines. (*a*) Golden mean case, (*b*) copper mean case. The right vertical axis in both figures represents frequency ν in GHz, whereas the left vertical axis represents Ω in kG.

respectively, where $\mathbf{T}_{left} = \mathbf{T}_{0n}(l_{0n} = l_{0m} = 0)$ and $\mathbf{T}_{right} = \mathbf{T}_{F_L+1m}(l_{F_L+1,m} = 0)$ are limit cases of the transfer matrices as defined in (2). Combining (4) with $\mathbf{M}_L$ gives the following condition for the eigenfrequencies for real boundary conditions: $(\mathbf{T}_{right}\mathbf{M}_{L\,left})_{11} = 0$. As expected, this condition can be satisfied only for discrete values of Ω. No recursion relation for the above 1,1 element is at hand; hence the full matrix relations (3) need to be used to obtain the eigenfrequencies.

In Figure 2 we compare the modes of the golden and copper mean lattices for real boundary conditions. The parameters are such that $l_{Am} = l_{Bn}$ and $l_{Bm} = l_{An}$. In this 'symmetric' case the total width of allowed bands in a quasi-continuous cluster of bands of the type shown in figure 1(*b*) exceeds the total width of gaps more pronouncedly than in the asymmetric case discussed above. For large numbers of blocks in the stack, the regions containing the discrete eigenvalues are almost identical with the allowed bands for the periodic boundary conditions. The only difference is that there are also some isolated eigenfrequencies independent on L when L is large enough. They usually correspond to surface modes (two of them are shown in figure 4) of properties similar to those of the recently reported isolated localised electronic states [9]. The character of all other modes varies throughout the frequency spectrum considerably, more in the case of the copper mean than in the case of the golden mean. The spectrum of the golden mean case forms a Cantor set and the spatial dependence of all the modes has a self-similar character. On the other hand, the spectrum of the copper mean case (figure 2(*b*)) appears to be composed of a mixture of self-similar and regular patterns. The latter are represented by a homogeneous dense cluster of eigenfrequencies extending from Ω = 3126.8 to 3166.2 G that is shown enlarged in figure 3(*a*). This cluster resembles the bands of extended modes of periodic systems. Also the spatial dependence of $|\psi_M(y)|$ in this frequency range is very regular (figure 4), and considerably resembles that of the extended modes of a periodic system. However, the sign of $\psi_M(y)$ in individual blocks varies aperiodically. In other frequency regions, the spectrum is Cantor-like (e.g. figure 3(*b*)) and the corresponding modes are neither fully localised nor extended. The 'Bloch-like' behaviour of some modes found here resembles that of the electronic states of the Thue–Morse crystal [8], which is not surprising in view of some similarity between the two cases mentioned above. This would suggest that both the Thue–Morse and the

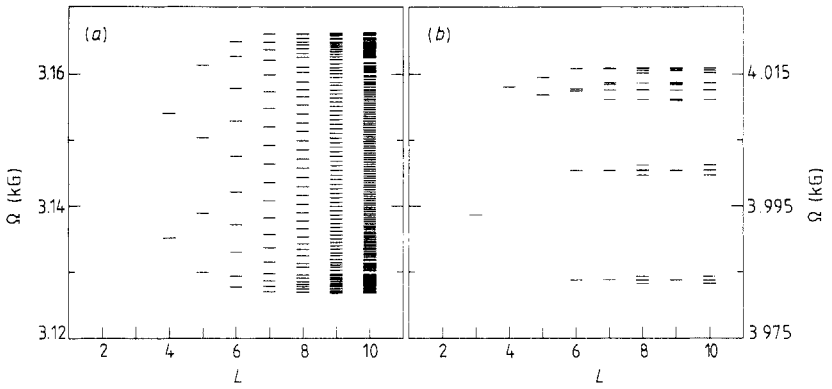

Figure 3. Two parts of the spectra of figure 2(*b*) enlarged.

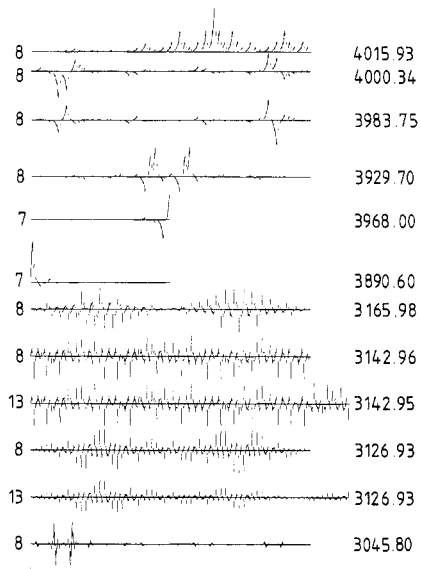

Figure 4. Several modes of the copper mean lattice spectra of figure 2(*b*). The plots represent $\psi_M(y)$ inside the magnetic films only. The horizontal line in each plot represents the zero of $\psi_M(y)$. Generation number is indicated on the left of each plot and the frequency on the right. The y scale is the same in each plot, so that only about $\frac{1}{30}$ of the whole extent of the stack with 13 generations (5461 blocks) is shown.

copper mean lattices are intermediate cases between periodic and quasi-periodic crystals, as was claimed in [8] for the Thue–Morse lattice. On the other hand the structure factor of one-dimensional systems arranged according to the Thue–Morse sequence seems to point somewhere between the quasi-periodic and random systems [21]. Our preliminary calculations suggest that this may also be true for the Fibonacci lattice with the copper mean. The exact classification of these structures deserves further attention.

From our model calculations, we have noticed here that there are important differences between the spectra of the golden mean and copper mean lattices. This suggests, among other things, that by purely geometrical means (ordering of layers), one is to a large extent able to modify the excitation spectra of the resulting systems. It will be

interesting to know if the calculated differences in the properties of generalised Fibonacci lattices are of importance from the experimental point of view.

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